EMPLOYING THE METHODS FROM THE THEORY OF SINGULARLY PERTURBED SYSTEMS TO AN ANALYSIS OF THE CONDITIONS UNDER WHICH A MAGNETIC FIELD IS FROZEN INTO AN ELECTRON FLUID

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An important place is occupied in plasma theory by the two-fluid magnetohydrodynamic model [1-3]. Such a model is particularly useful in studying the motions of a plasma in crossed $\mathbf{E}\times\mathbf{H}$ fields. For this particular case there exists a sufficiently reliable basis for the applications of hydrodynamic equations and a number of solutions have been derived for nonlinear wave motions. The majority of the solutions for the nonlinear equations of two-fluid magnetohydrodynamics is based on the utilization of such simplifying assumptions as the properties of quasineutrality ($n_e = n_i$) and the "freezing-in" concept (n_e/B = const), whose validity, as a rule, is not analyzed. A sufficient criterion for the validity of approximating quasineutrality is satisfaction of the inequality $\mu \ll 1$ ($\mu = \omega_{Be}/\omega_{p_e}$, where $\omega_{Be} = |e|B_0/m_e c$ is the electron cyclotron frequency, while $\omega_{p_e} = \sqrt{4\pi e^2 n_0/m_e}$ is the Langmuir frequency). In the opposite limit case ($\mu \gg 1$) we make use of the "frozen-in" condition.

By turning to the properties of quasineutrality, i.e., the concept of being "frozen in," makes it possible to lower the order of magnitude of the system of differential equations. In this event, the terms with small parameters for the higher derivatives are necessarily excluded. From the mathematical standpoint, any attempt to take into consideration these excluded terms signifies the introduction of singular perturbations, to which our attention was first drawn in [4, 5]. It was demonstrated in [6-10] that quasilinearity cannot be regarded as a universal property of a plasma when $\mu \ll 1$: the field of charge separation may lose stability and oscillations may develop within the plasma, the amplitudes of these oscillations corresponding virtually to the complete separation of the charges on scales considerably smaller than that of the external perturbations. The developing steady oscillations may significantly affect the nature of the change in the slow variables describing the plasma [4, 6, 8]. Thus, a detailed analysis of the equation for variables with the least scale change must precede both the analytical and numerical solution of the hydrodynamic equations of the plasma.

In the present study we utilized the method from the theory of singular perturbations of the systems to analyze the properties of frozen-in electrons (n_e/B = const) for a plasma in which $\mu \gg 1$. It is our primary goal to propose a mathematical apparatus which would make it possible to derive steady-state solutions for the case in which the frozen-in condition need not be satisfied, whereas the density and velocity of the electrons in a direction perpendicular to the magnetic field and to the direction of wave propagation execute oscillations at a frequency on the order of the electron cyclotron frequency.

Let us examine a straight magnetosonic wave of finite amplitude. We will describe this wave within the framework of the equations of two-fluid collision-free magnetohydrodynamics, with a condition of applicability for these equations being the satisfaction of the relationship $\ell \gg \rho$ (ℓ is the characteristic scale of the spatial change in the hydrodynamic quantities, with ρ representing the Larmor electron radius) [11]. We will use a coordinate system whose Oz axis is directed along the magnetic field $\mathbf{B}$. We assume that all of the quantities depend exclusively on the coordinate x, and that the magnetosonic wave is propagated along the Ox axis. A wave-type solution is sought in its dependence on the variable $\zeta = x - ut$. The system of Maxwell equations in conjunction with the equations of motion and continuity for electrons and ions is then written as follows:

$$E_y = (u/c)(B - B_0); \quad (1)$$

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$$dE_x/d\zeta = 4\pi|e|(n_i - n_e); \quad (2)$$

$$-\gamma^2 dB/d\zeta = 4\pi|e|(n_i v_{iy} - n_e v_{ey})/c; \quad (3)$$

$$m_{e,i} n_0 u dv_{e,ix}/d\zeta = dp_{e,i}/d\zeta \pm |e| n_{e,i}(E_x + v_{e,iy}B/c); \quad (4)$$

$$m_{e,i} n_0 u dv_{e,iy}/d\zeta = \pm |e| n_{e,i}(E_y - v_{e,ix}B/c); \quad (5)$$

$$n_{e,i} = n_0 u/(u - v_{e,ix}) \quad (6)$$

[n_0 and B_0 are the unperturbed values of particle density and the magnetic field, $\gamma = \sqrt{1-(u/c)^2}$]. It is assumed in (1)-(6) that in the unperturbed state as $\zeta \to +\infty$ there are no hydrodynamic motions: $v_{e,i}(+\infty) = 0$. Nevertheless, no consideration is given to deviations from equilibrium as $\zeta \to +\infty$, resulting from the thermal noise of the intrinsic plasma oscillations. As in [11, 12], in Eqs. (1)-(6) we neglect the phenomena associated with the finiteness of the Larmor radius of the charged particles (magnetic viscosity), since $\ell \gg \rho$, as well as collisional friction. Applying to system (1)-(6) the procedure described in [8], for the case of $\mu \gg 1$ we reduce it to the following:

$$(M\sqrt{\varepsilon}/\mu)dY/d\xi = F(X, Y, b, Z)/Q(X, Y, b, Z); \quad (7)$$

$$(M\sqrt{\varepsilon}/\mu)dZ/d\xi = b - \tilde{n}; \quad (8)$$

$$M\sqrt{\varepsilon}\mu dX/d\xi = Y; \quad (9)$$

$$(\gamma^2/M\sqrt{\varepsilon}\mu)db/d\xi = Z(\tilde{n} + \varepsilon n) - Xn\varepsilon\mu^2, \quad (10)$$

where $\xi = \zeta\omega_{pe}^2/c\omega_{Be}$, $X = E_x/M\sqrt{\varepsilon}\mu B_0$, $b = B/B_0$, $n = n_i/n_0$, $\tilde{n} = n_e/n_0$ $(\tilde{n} = n - Y)$, $M = u/v_A$ $(v_A^2 = B_0^2/4\rho n_0 m_i)$, $\varepsilon = m_e/m_i$, $Q(X, Y, b, Z) = 1 - \beta\tilde{n}^2$, $\beta = T_e/m_e u^2$, $F(X, Y, b, Z) = -X[\tilde{n}^3 + \varepsilon Q(X, Y, b, Z)n^3(1 + b\varepsilon\mu^2)] - bZ[\tilde{n}^3 - \varepsilon^2 Q(X, Y, b, Z)n^3]$. In this case, system (7)-(10) is enhanced with the following algebraic equation to link the variables n and $\tilde{n}$ [8]:

$$\varepsilon(1 - 1/\tilde{n}) + 1 - 1/n = (\tilde{n} - 1)\varepsilon\beta + [\gamma^2(b^2 - 1) - M^2\varepsilon\mu^2X^2]/2M^2. \quad (11)$$

A single assumption was made in the derivation of (7)-(11): $T_i = 0$, $T_e = \text{const}$, and in view of the limitations imposed on system (1)-(6) we will subsequently assume that $\beta \ll 1$.

The solution of system (7)-(11) is sought for $\xi \to -\infty$ under additional conditions. For the variables b and X we have the boundary conditions in the region $\xi \to +\infty$, which has not yet been reached by the magnetosonic wave: $b(+\infty) = 1$, $X(+\infty) = 0$. As regards the variables Z and Y, out of physical considerations we require as $\xi \to +\infty$ only the proximity of the quantities $Z(+\infty)$ and $Y(+\infty)$ to their equilibrium values: $Z_{equ} = 0$ and $\tilde{n}_{equ} = b_{equ} = 1$. A more precise definition of these quantities is not possible. In the algebraic expression (11), which can be treated as the equation for ion density, transition to the unperturbed state $n(+\infty) = 1$ on consideration that $\beta \ll 1$ and $\varepsilon \ll 1$, corresponds to the root $n = 2M^2/[2M^2 + \gamma^2(1 - b^2) + M^2\varepsilon\mu^2X^2]$. Let us limit ourselves to a case of ways not overly powerful, for which the Alfvén–Mach number $M \leq 2.76$ [3] and the phenomena of "raking" or "reflection" of the ions do not occur. In this case, the totality of equations (7)-(11) with the additional conditions as $\xi \to +\infty$ relates to the class of singularly perturbed systems [13, 14], since in system (7)-(10) the derivatives have the small parameters $M\sqrt{\varepsilon}/\mu \ll M\sqrt{\varepsilon}\mu < 1$ (we note that $\mu \gg 1$, but in view of $u/c < 1$, $M\sqrt{\varepsilon}\mu < 1$).

In general outlines system (7)-(11) coincides with the corresponding equations from [4, 10], derived for $\mu \ll 1$. Unlike [4, 10] when $\mu \gg 1$, proceeding from the requirement of representativeness for system (7)-(10) (the possibility of expressing the fast variables in terms of the slow when the small parameter formally tends to zero [13, 14]), it is essential that we treat the equations for Y and Z as the fast subsystem with the smallest parameter for the derivatives, whereas in the case in which $\mu \ll 1$ the small parameter for the derivative appears in the equations for X and Y. The second fundamental difference from [4, 10], i.e., the equation for the magnetic field, is not separated out. For notation of the analyzed equations in standard form [15] under the condition of boundedness for the derivatives of the slow variables, it is necessary to use the variable Z (10), rather than $\alpha = db/d\xi$, as in [4, 6-10].*

In analyzing Eqs. (7)-(11) by the method from the theory of singularly perturbed systems [13, 14] it is necessary, first of all, to examine the degenerate system which is derived if in (7)-(11) it is formally assumed that $M\sqrt{\varepsilon}/\mu = 0$ and if we assume that $M\sqrt{\varepsilon\mu} \neq 0$:

$$F(\bar{X}, \bar{Y}, \bar{b}, \bar{Z}) = 0, \ (Q(X, Y, b, Z) \neq 0); \tag{12}$$

$$\bar{\bar{n}} - \bar{b} = 0; \tag{13}$$

$$M\sqrt{\varepsilon\mu}d\bar{X}/d\xi = \bar{Y}; \tag{14}$$

$$(\gamma^2/M\sqrt{\varepsilon\mu})d\bar{b}/d\xi = \bar{Z}\bar{\bar{n}} - \bar{X}\bar{n}\varepsilon\mu^2; \tag{15}$$

$$\bar{n} = 2M^2/[2M^2 + \gamma^2(1 - \bar{b}^2) + M^2\varepsilon\mu^2\bar{X}^2]. \tag{16}$$

For the sake of simplicity of exposition, we have dropped the terms in (12)-(16) which contain the parameter ε, since $\varepsilon \ll M\sqrt{\varepsilon\mu} < 1$. According to (13), the frozen-in condition corresponds to transition from system (7)-(11) to the degenerate system (12)-(16), which yields all of the known results on the structure of magnetohydrodynamic shock waves [12]. Indeed, assuming that $\bar{X} = -(d\varphi/d\xi)/M\sqrt{\varepsilon\mu}$ and taking into consideration that $(u/c)^2 \ll 1$, it is easy to derive from (12)-(16) the equation for φ:

$$d^2\varphi/d\xi^2 = \varphi + 1 - \bar{n}, \tag{17}$$

which in its external form coincides with the corresponding equation from [12], derived under the assumptions that $\mu \gg 1$, $u/c < 1$. However, in (17) $\bar{n} \approx 2M^2/[1 + 2M^2 - (1 + \varphi)^2]$, while in [12], owing to the oversimplification of the equations of ion motion, the density of the ions is expressed somewhat differently which, by the way, has no significant effect on the behavior of the function $\varphi = \varphi(\xi)$. It follows from an analysis of Eq. (17) that with a change in ξ from $+\infty$ to $-\infty$ the sign of $\bar{X}$ changes from + to − [12].

The possibility of utilizing (12)-(16) in the place of (7)-(11) depends on the asymptotic stability of the connected system [13, 14] in the vicinity of the chosen root ($\bar{Z} = \bar{Z}(\bar{X}, \bar{b})$; $\bar{Y} = \bar{Y}(\bar{X}, b)$). We will write the connected system

$$(M\sqrt{\varepsilon}/\mu)d\bar{\bar{Y}}/d\xi = F(\bar{X}, \bar{\bar{Y}}, \bar{b}, \bar{\bar{Z}})/Q(\bar{X}, \bar{\bar{Y}}, \bar{b}, \bar{\bar{Z}}); \tag{18}$$

$$(M\sqrt{\varepsilon}/\mu)d\bar{\bar{Z}}/d\xi = \bar{b} - (\bar{n} - \bar{\bar{Y}}), \tag{19}$$

in which the variables $\bar{X}$ and $\bar{b}$ are treated as parameters. Asymptotic stability occurs if the roots of the system $F(\bar{X}, \bar{Y}, \bar{b}, \bar{Z}) = 0$ and $\bar{b} - \bar{\bar{n}} = 0$, corresponding to the equilibrium point ($\bar{Z} = \bar{Z}(\bar{X}, \bar{b})$, $\bar{Y} = \bar{Y}(\bar{X}, \bar{b})$) of the connected system (18), (19) as $\xi \to -\infty$ exhibit the properties of a stable node or focus. The equilibrium point ($\bar{Z}$, $\bar{Y}$) serves as the center. In this connection, we will use the results from [16], where the condition of asymptotic stability for system (18), (19) is replaced by the condition of stability under the addi-

*The behavior of fast variables affect the nature of the change in the slow variables [13, 14] and according to the procedure for the solution of singularly perturbed equations [18], in the oscillatory course of change in the fast variables, these are replaced by certain averaged quantities. As the fast variables it is natural to select such physical quantities which can be measured locally in the experiment. For the slow variables we should choose those which are measured by the averaged quantities. In this case, the results of the theory can be compared with the experiment. Moreover, if the point of equilibrium for the fast variables turns out to be unstable [19], then in the presence of limiting curves (only then it is possible to construct a solution) and of a possible solution with these curves must system (7)-(10) remain representative [13, 14] and the variables chosen at the initial instant as the slow variables must, on reaching the limited curves, remain precisely such. According to [4, 7, 10], when $\mu \ll 1$ there exists a singular multiplicity $Q = 0$ which, as was demonstrated in [10], exhibits attraction properties. Consequently, selection as the slow variables (b, α) is improper and we should take a look at the variables (b, Z). As a result, the criterion of asymptotic stability, derived in [10] to satisfy the conditions of quasineutrality ($n_e = n_i$) has been satisfied for the case in which $Z > 0$, i.e., when $db/d\xi > 0$.

tional assumption of the existence and positiveness of the mean of the derivative Lyapunov function (we will seek the solution as $\xi \to -\infty$), where the mean is calculated along the integral curves of system (18), (19). In the system (7)-(10) we introduce the slow coordinates $\tau = \xi\mu/M\sqrt{\varepsilon}$ and carry out the substitution of the variables $Y = Y_0 + u$, $Z = Z_0 + v$, where $(Y_0; Z_0)$ is the root of system (12), (13). As a result, with accuracy to terms on the order of $1/\mu^2$, ε and β we obtain

$$du/d\tau = -vb(b-u)^3 - \varepsilon n^3 X[1-((b-n)/b)^3] - \\ -(Y_0+u)(dn/dX)/\mu^2 + (1-dn/db)(-X/b+v)(b-u)(M^2\varepsilon/\gamma^2) - \\ -\beta v\Phi(u,\ b,\ X); \quad (20)$$

$$dv/d\tau = u + (Y_0+u)/\mu^2 b - (M^2\varepsilon/\gamma^2)(X/b^2)(v - X/b)(b-u); \quad (21)$$

$$dX/d\tau = (n - b + u)/\mu^2; \quad (22)$$

$$db/d\tau = (M^2\varepsilon/\gamma^2)(v - X/b)(b-u). \quad (23)$$

Here $\Phi(u,\ b,\ X)$ is some function whose specific form is unimportant, since it is not included in the criterion of stability, as will become evident later on. The Lyapunov function $\mathcal{V}_0$ is introduced on the basis of the integral of motion from system (20)-(23) in approximation of $\varepsilon = \beta = 1/\mu^2 = 0$:

$$\mathcal{V}_0 = (b/2)v^2 - (b-2u)/2(b-u)^2 + 1/2b.$$

We differentiate $\mathcal{V}_0$ on the strength of system (20)-(23):

$$d\mathcal{V}_0/d\tau = \varkappa(u,\ v,\ b,\ X) \\ (\varkappa(u,\ v,\ b,\ X) = (Y_0+u)v/\mu^2 - \beta v\Phi(u,\ b,\ X)u/(b-u)^3 - \\ -(\varepsilon M^2/\gamma^2)[v(X/b)(v-X/b)(b-u) + n^3X(b^3-(b-u)^3)(u/(b-u)^3)/b^3M^2 + (n^2b-1)(v - \\ -X/b)(b-u)(u/(b-u)^3) - XY_0 n^2 u/(b-u)^2M^2]). \quad (24)$$

Expression (24) has been derived in the first nonvanishing approximation, with accuracy to terms on the order of ε, β, and $1/\mu^2$. According to [16], the solution of system (7)-(10) is close to the solution of system (12)-(16) for satisfaction of the inequality

$$L = \int_0^T \varkappa(u,\ v,\ b,\ X)\,d\tau > 0 \qquad (T \gg 1). \quad (25)$$

The integral in (25) is calculated for fixed b and X along the solution of system (20), (21). It is possible approximately to find L on the basis of the fact that the resting point of system (18), (19) is the center and, therefore, assuming $u \approx |a|\sin\tau$, $v \approx -|a|\cos\tau$ ($|a|$ is the amplitude for the change in the fast variables, and $|a| \ll 1$), so that with accuracy to the terms $\sim a^2$ we have

$$L = -(M^2\varepsilon/\gamma^2)(X/b)(a^2/2)(1 + 8M^4(bn+1)/nb). \quad (26)$$

Condition (25) guarantees proximity of the solution of system (7)-(10) to the solution of the degenerate system (12)-(16), when the parameters $M\sqrt{\varepsilon}$ and $1/\mu$ are of identical order of magnitude [16]. According to (25), (26) approximation of the condition of being frozen in is valid for $X < 0$ [it follows from (12)-(16) that this is a "descending" segment b as $\tau \to -\infty$). When $X > 0$ this approximation, generally speaking, may prove to be invalid and will require additional analysis. Figure 1 shows the phase plane (Z, Y), qualitatively mapping the principal singularities of the fast subsystem (7), (8) for $\beta b^2 \ll 1$. In its physical sense the motion along the curve $F(X,\ Y,\ b,\ Z) = 0$ (line 1) corresponds to the drifting nature of the y component of the electron flux. The curve $Q(X,\ Y,\ b,\ Z) = 0$ defines the unique set in which the change in sign occurs for the right-hand side of the equation for Y, and here the thermal electron flux is compared to the hydrodynamic flow: $n_e v_{T_e} = n_0 u$ ($v_{T_e} = \sqrt{T_e/m_e}$) and the maximum possible synchronicity in particle oscillation is attained. The unique set $Q(X,\ Y,\ b,\ Z) = 0$ plays the same role in the singularly perturbed systems as do the stable roots of the degenerate equations [5]. The dashed line 4 in the figure represents the trajectory of the system, 5 is the line on which the condition for the "frozen-in" concept has been satisfied ($\tilde{n} = b$), and line 3 represents $\tilde{n} = 0$.

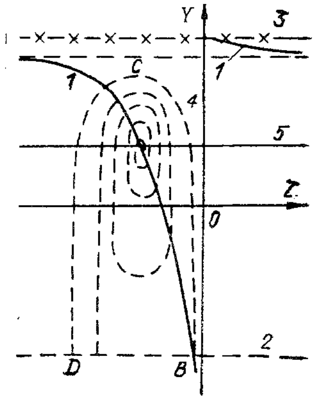

Fig. 1

In the degenerate system (12)-(16), since we can indicate no other algebraic roots exhibiting the properties of stability in the region X > 0 toward which the solution might strive, analysis at the phase plane of all singularities of system (7)-(10) is necessary. The motion of the integral curve of subsystem (7), (8) is determined by three regimes, depending on the relationship between $M\sqrt{\varepsilon}$ and $1/\mu$. Near the position of equilibrium the motion is described in accordance with the theory developed in [16]. With increasing distance from the point (Y, Z) it is bounded from below by the unique set Q(X, Y, b, Z) = 0 (curve 2). If during the time for change in the slow variables (9), (10), when the sign of X does not change, the integral curve reaches the unique set Q(X, Y, b, Z) = 0, and in this case, according to [5], the motion proceeds along curve 2, according to (8), where Y belongs to Q(X, Y, b, Z) = 0. The single valuedness of behavior will be disrupted at the point of intersection between curves 2 and 1, which is the saddle. After its passage, the unique set Q(X, Y, b, Z) = 0 becomes unstable. Out of physical considerations, having chosen the direction of motion toward increasing Y (since as $\bar{Y} \to -\infty$, $n_e \to \infty$), we can prove that the trajectory executes an encirclement of the point $(\bar{Y}, \bar{Z})$ and once again reaches curve 2, i.e., the motion is repeated. From an analysis of the phase portrait of the system depicted in Fig. 1, we can see that the limit trajectory of the fast subsystem (7), (8) is independent of the initial perturbations and thus exhibits stability. Indeed, if the initial value is removed from $(\bar{Y}, \bar{Z})$ over a considerable distance such as, for example, to the first quadrant of the plane (Y, Z), then in the direction $Y \to +\infty$ the motion is forbidden, since as $\tilde{n} \to \varepsilon^{1/3}\tilde{n}$ ($\tilde{n} = n - Y$) in (7) the quantity F(X, Y, b, Z) is now of order ε. The motion along OY in this case is curtailed and can proceed only along the OZ axis until the integral curve intersects curve 1. The motion will then proceed in the direction $Y \to -\infty$ and the integral curve reaches curve 2. It is precisely the reaching by any trajectory of the unique set Q(X, Y, b, Z) = 0 and the motion along that trajectory to the point of intersection with F(X, Y, b, Z) = 0 that ensures stability of the limit trajectory. [The initial values must be positioned on the phase plane above curve 2, since below this curve Eqs. (1)-(6) are insufficient: here we must take into consideration the viscous terms of the pressure tensor.] The derived pattern of change in the fast variables is close to the relaxation oscillations [17]. Here we find oscillations in the concentration of electrons $\tilde{n} = n - Y$ and in the y-component of electron velocity: $v_{ey} \sim Z$. It is not difficult to estimate the period of these oscillations. The limit trajectory on the (Y, Z) plane consists of the segment DB of the unique set Q(X, Y, b, Z) = 0 along which it moves through the characteristic time T_1, and the curve BCD, encompassing the point $(\bar{Y}, \bar{Z})$, starting from the point of intersection for curves 1 and 2 and reaching the unique set Q(X, Y, b, Z) = 0 after encircling point $(\bar{Y}, \bar{Z})$. The system passes through this curve within the characteristic time T_2. Using (8) and taking into consideration that $\zeta = x - ut$, let us estimate T_1 and T_2. In motion over segment BCD of the limit trajectory we can assume that $\beta\tilde{n}^2 \ll 1$, and therefore from (18) and (19) in the $\varepsilon = 0$ approximation we have

$$dY^*/dZ^* = -\bar{b}Z^*(\bar{b} - Y^*)^3/Y^* \qquad (Y^* = \bar{b} - \bar{n} + \bar{\bar{Y}},\ Z^* = \bar{\bar{Z}} + \bar{X}/\bar{b}). \tag{27}$$

Integrating (27) and assuming for the sake of simplicity that $\bar{b} \simeq 1$, we obtain

$$Z^{*2} + (Y^*/(1 - Y^*))^2 = A^2 \ (A = \text{const}). \tag{28}$$

In view of the fact that $\bar{\bar{Y}} < \bar{n}$, from (28) it is easy to find the limitation imposed on $|A|$: $|A| < 1$. Hence it follows that on motion over the limit trajectory on the BCD segment (see Fig. 1) $Y^* < 1/2$ and $Z^* < 1$, i.e., $n_e \geq B/2$, while the maximum oscillation amplitude of the y-component of the electron velocity $|v_{ey}| \leq u$. In view of the foregoing we have $T_2 \sim 1/\omega_{Be}$. In motion over the segment DB ($\tilde{n} = 1/\sqrt{\beta} \gg 1$, where $\beta \ll 1$) $\tilde{n} \gg b$, so that consequently $T_1 \sim v_{T_e}/u\omega_{Be}$. Since $\beta \ll 1$, then $T_1 \ll T_2$, and the period of the resulting oscillations is therefore $T = T_1 + T_2 \simeq 1/\omega_{Be}$.

Thus, when $\mu \gg 1$ we are confronted with the "bunching" of the electrons at the Larmor radius, with a buildup of relation-type cyclotron oscillations, since motion along the unique set in this case may be regarded as a discontinuity in the y-component of the electron velocity, given a constant concentration of these electrons. It should be stressed that it is the electron density and the y-component of the electron velocity that execute regular oscillations at the front of the wave. More complex is the intermediate regime in which, given a change in the slow variables (9), (10), the sign of X changes until the integral curve of the fast subsystem (7), (8) is capable of reaching the unique set $Q(X, Y, b, Z) = 0$. In this case, it executes motion in the region between the equilibrium point $(\bar{Y}, \bar{Z})$ and the limit trajectory, without ever reaching the latter. Realization of such motion occurs, apparently, when between $M\sqrt{\varepsilon}$ and $1/\mu$ the relationship $M\sqrt{\varepsilon}\mu \ll 1$ is satisfied.

Since the fast variable (Y, Z) change in the bounded region, then as their change takes place near the equilibrium point $(\bar{Y}, \bar{Z})$ or as they reach the limit trajectory, in the equations for (X, b) we can carry out the averaging procedure. In the first case, the averaging is accomplished in complete agreement with [16]. In the second case, we are confronted with the singularity that is associated with the fact that on reaching the set $Q(X, Y, b, Z) = 0$ the derivatives of the fast variables have not been determined, so that there is no smoothness in the right-hand sides in the equations for (X, b). However, even here the averaging procedure is valid [18]. After averaging of the equations for the slow variables (9), (10), we obtain

$$M\sqrt{\varepsilon}\mu d\langle X\rangle/d\xi = \langle Y\rangle; \tag{29}$$

$$(\gamma^2/M\sqrt{\varepsilon}\mu)d\langle b\rangle/d\xi = \langle Zn\rangle - \langle ZY\rangle - \varepsilon\mu^2\langle Xn\rangle, \tag{30}$$

where $\langle A\rangle$ denotes the average value of A, and $\langle A\rangle$ depends on the regime dependent on $M\sqrt{\varepsilon}$ and $1/\mu$ that is established in the fast subsystem (7), (8). Since $\langle Y\rangle$, $\langle Zn\rangle$, $\langle ZY\rangle$, and $\langle Xn\rangle$ are functions of $\langle X\rangle$ and $\langle b\rangle$, system (29), (30) describes the change in the new slow variables ($\langle X\rangle$, $\langle b\rangle$) which are close to (X, b) in the first case [16, 18]. In the second case, the solution of system (29), (30) can, however, differ from the solution for system (12)-(16) both in terms of the scale of change in the slow variables, and in the functional relationship.

Thus, for purposes of investigating the regions of applicability for such characteristic properties of the plasma as the "frozen-in" concept and quasineutrality, it is essential that we resort to the theory of singular perturbations [13, 14]. The unique feature of the hydrodynamic equations of a plasma as an object of the theory of singularly perturbed systems, where the position of the equilibrium for the corresponding connected system in zeroth approximation of the small parameters turns out, as a rule, to be the center, and therefore requires a more careful analysis of its stability, is based on construction of the Lyapunov functions [16]. The oscillatory nature of the changes in the fast variables demands the procedure of averaging in solving the system of equations for the slow variables [18]. With a stable motion of the fast variables about the position of equilibrium the average values of the slow variables will be no different from those which are derived in replacement of the connected system by the degenerate system. As was demonstrated by the analysis conducted in [10, 5] and in the present study, in the case of instability the fast variables change in the bounded region of the phase space and even in the simplest model of a plasma with "cold" electrons ($v_{T_e} < u$) and where the effect of the finite Larmor radius is neglected it is possible to point to the limit trajectory asymptotically attained by the integral curve of the fast subsystem. The existence of a limit trajectory is ensured, first of all, by the unique set $Q = 0$ exhibiting attraction properties [5]. The presence of a unique set not only guarantees stability of the limit trajectory for the fast variables, but is determining in the selection of the slow variables which must necessarily be represented in standard form [15] in order for the averaging procedure to make any sense when $Q = 0$.

Analysis of the fast variables in the equations of two-fluid magnetohydrodynamics by the method from the theory of singularly perturbed systems makes it possible asymptotically to describe with precision the nonlinear electrostatic oscillations which exhibit a relaxational nature [17, 19] owing to the existence of the unique set Q = 0. The change in the electron concentration and in the y-component of the electron velocity acquires the nature of regular relaxation-type cyclotron oscillations with a rather substantial change in electron concentration so that it becomes possible, therefore, to speak of the "bunching" of both the final phase of the oscillatory process and limit values for electron concentrations determined by the limiting curves Q = 0 and $n_e = 0$.

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LITERATURE CITED

1. B. B. Kadomtsev, Collective Phenomena in a Plasma [in Russian], Nauka, Moscow (1976).
2. R. Z. Sagdeev, Collective Processes and Shock Waves in a Rarefied Plasma [in Russian], Vol. 4, Atomizdat, Moscow (1964).
3. Yu. A. Berezin, Numerical Studies of Nonlinear Waves in a Rarefied Plasma [in Russian], Nauka, Novosibirsk (1977).
4. G. V. Sholin and S. A. Trushin, "Stationary deviations from quasineutrality in plasma dynamics," in: Proceedings, 17th International Conference on Phenomena in Ionized Gases, Budapest, Hungary (1985).
5. M. M. Khapaev and G. V. Sholin, "Analysis of the hydrodynamic plasma equations by the methods from the theory of singular perturbations," Dokl. Akad. Nauk SSSR, 286, No. 3 (1986).
6. S. A. Trushin and G. V. Sholin, "The development of Langmuir oscillations in the motion of a plasma across a magnetic field," Dokl. Akad. Nauk SSSR, 255, No. 5 (1980).
7. S. A. Trushin and G. V. Sholin, "The self-oscillatory regime for the excitation of stochastic electrostatic waves in the motion of a plasma across a magnetic field," Dokl. Akad. Nauk SSSR, 261, No. 3 (1981).
8. S. A. Trushin and G. V. Sholin, "The self-oscillating regime for the excitation of electrostatic noise and the dynamics of dissipative processes at the front of transverse magnetosonic waves of finite amplitude," Preprint, No. 3449/6, Akad. Nauk SSSR, IAÉ, Moscow (1981).
9. S. K. Zhdanov and B. A. Trubnikov, "A magnetosonic rarefaction soliton and the generation of high-frequency oscillations in the dynamic development of a z pinch," Zh. Éksp. Teor. Fiz., 83, No. 6 (1982).
10. S. A. Trushin, G. V. Sholin, and M. M. Khapaev, "The nature of the change in an electric charge-separation field at the front of nonlinear magnetosonic waves," Fiz. Plazmy, 14, No. 1 (1988).
11. L. A. Artsimovich and R. Z. Sagdeev, Plasma Physics for Physicists [in Russian], Atomizdat, Moscow (1979).
12. K. Longmayer, Plasma Physics [Russian translation], Atomizdat, Moscow (1966).
13. A. N. Tikhonov, "Systems of differential equations containing small derivative parameters," Matem. Sb., 31(93), No. 3 (1952).
14. A. B. Vasil'eva and V. F. Butuzov, Asymptotic Expansions for the Solutions of Singularly Perturbed Equations [in Russian], Nauka, Moscow (1973).
15. N. N. Krylov and N. N. Bogolyubov, New Methods in Nonlinear Mechanics [in Russian], GTTTs, Kiev (1934).
16. M. M. Khapaev, "The A. N. Tikhonov theorem for singularly perturbed systems," Dokl. Akad. Nauk SSSR, 271, No. 5 (1983).
17. E. F. Mishchenko and N. Kh. Rozov, Differential Equations with a Small Parameter and Relaxation Oscillations [in Russian], Nauka, Moscow (1975).
18. M. M. Khapaev, "On an averaging method and on certain problems associated with averaging," Differents. Uravn., 2, No. 5 (1966).
19. A. A. Andronov, A. V. Vitt, and S. Z. Khaikin, The Theory of Oscillations [in Russian], Nauka, Moscow (1977).